Calculus Warm Up #10-1

State the domain, find any relative extrema, points of inflection and concavity, then accurately sketch the graph.

$y = x - \ln x$

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dom: $x > 0$ (can't take ln of a negative)

$y = x - \ln x$

$y' = 1 - \frac{1}{x}$ undef @ $x = 0$

$0 = \frac{x-1}{x}$

$x = 1$

y': 0.5, 2

0, 1

$-$, $+$

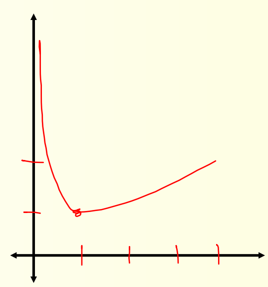
Min @ $(1, 1)$

$y = 1 - \ln 1$

$1 - 0$

$y'' = \frac{1}{x^2}$ undef $x = 0$

outcomes always positive so, concave up



7.8

- Indeterminate forms

- L' Hôpital's Rule

From Ch. 2, when:

$$\lim_{x \to c} f(x) = \frac{0}{0} \text{ or } \frac{\infty}{\infty}$$

We could not determine the limit so we fussed with the function to change its form allowing us to evaluate the limit.

Ex:

$a^2 - b^2$

$$\lim_{x \to 0} \frac{e^{4x} - 1}{e^{2x} - 1} = \frac{e^0 - 1}{e^0 - 1} = \frac{1-1}{1-1} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\cancel{(e^{2x} - 1)}(e^{2x} + 1)}{\cancel{(e^{2x} - 1)}}$$

$$\lim_{x \to 0} e^{2x} + 1$$

$$e^0 + 1$$

$$1 + 1$$

$$\boxed{2}$$

If direct substitution gives you an $\underbrace{\text{indeterminate form}}_{\frac{0}{0} \text{ or } \frac{\infty}{\infty}}$, then try L'Hôpital's Rule:

THEOREM 7.15 L'HÔPITAL'S RULE

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. If the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form $0/0$ or ∞/∞, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists (or is infinite).

Evaluate

$$\lim_{x \to 0} \frac{e^{2x} - 1}{x}. = \frac{e^0 - 1}{0} = \frac{1-1}{0} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\frac{d}{dx}[e^{2x} - 1]}{\frac{d}{dx}[x]} \leftarrow \text{(Applying L'Hôpital's Rule}$$

$$\lim_{x \to 0} \frac{2e^{2x}}{1}$$

$$= 2e^0 = \boxed{2}$$

EXAMPLE 2 Indeterminate form ∞/∞

Evaluate

$$\lim_{x\to\infty} \frac{\ln x}{x}. = \frac{\ln \infty}{\infty} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{d}{dx}[\ln x]}{\frac{d}{dx}[x]} \Rightarrow \lim_{x\to\infty} \frac{\frac{1}{x}}{1}$$

$$= \frac{1}{\infty} = \boxed{0}$$

EXAMPLE 3 Applying L'Hôpital's Rule more than once

Evaluate

$$\lim_{x\to-\infty} \frac{x^2}{e^{-x}}. = \frac{(-\infty)^2}{e^{\infty}} = \frac{\infty}{\infty}$$

$$= \lim_{x\to-\infty} \frac{2x}{-e^{-x}} = \frac{2(-\infty)}{-e^{\infty}} = \frac{-\infty}{-\infty}$$

$$= \lim_{x\to-\infty} \frac{2}{e^{-x}} = \frac{2}{\infty} = \boxed{0}$$

EXAMPLE 4 Indeterminate form $0 \cdot \infty$

(Can only apply L′Hôpital's rule for $\frac{0}{0}$ or $\frac{\infty}{\infty}$)

Evaluate

$$\lim_{x \to \infty} e^{-x}\sqrt{x}. = (e^{-\infty})\sqrt{\infty}$$

$$\left(\frac{1}{e^{\infty}}\right)\sqrt{\infty} \quad = 0 \cdot \infty$$

Rewrite

$$\lim_{x \to \infty} \frac{\sqrt{x}}{e^x} = \frac{\infty}{\infty} \checkmark$$

$$\lim_{x \to \infty} \frac{\frac{1}{2}x^{-1/2}}{e^x} \longleftarrow \text{L′Hôpital's Rule}$$

$$\lim_{x \to \infty} \frac{1}{2\sqrt{x}\,e^x} = \frac{1}{2(\infty)(\infty)} = \frac{1}{\infty} \quad \boxed{0}$$

Summary:

Indeterminate Forms

0^0 $\quad\infty^0\quad$ 1^∞	Use Natural Logarithms
$\frac{0}{0}$ $\quad$ $\frac{\infty}{\infty}$	Use L'Hôpital's Rule
$\infty - \infty$	Change the form by adding or subtracting
$0 \cdot \infty$	Change the form to get $\frac{0}{0}$ or $\frac{\infty}{\infty}$ and then use L'Hôpital's Rule

Determinate Forms

$\infty + \infty = \infty$ $\qquad$ $-\infty - \infty = -\infty$

$0^{\infty} = 0$ $\qquad$ $\frac{1}{0^{\infty}} \leftarrow 0^{-\infty} = \infty$

$\frac{0}{L}$ $\qquad$ $\frac{L}{\pm\infty}$ $\qquad$ $\frac{0}{\pm\infty}$ $\qquad$ Limit is Zero

$\frac{L}{0}$ $\qquad$ $\frac{\pm\infty}{L}$ $\qquad$ $\frac{\pm\infty}{0}$ $\qquad$ Limit is Infinite

HW: p. 413

1 - 29 odd

* Turn in #5 tomorrow

HW quiz tomorrow
p. 369 (2 days),
p. 392 (2 days)

Answers #5:

4a) Show clear process

b) show derivative $= 0$ when $x = 3$

y-coord of $P = 2$

c) $\frac{d^2y}{dx^2} = \left[(8y-3x)(3\frac{dy}{dx}-2)-(3y-2x)(8\frac{dy}{dx}-3)\right] / (8y-3x)^2$

plug in $x = 3$, $y = 2$, $\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2}$ @ $(3,2)$ $= -\frac{2}{7}$ concave down confirms Max @ $(3,2)$

5a) Show clear process

b) $\frac{1}{2} = \frac{y}{2y-x}$

$\downarrow$

$x = 0$

points: $(0, \pm\sqrt{2})$

c) Set $\frac{dy}{dx} = 0$

$\downarrow$

$y = 0$

but $y \neq 0$ in original (show this)

d) Find the related rates equation.

Find x when $y = 3$ in the original.

Plug in everything you know...

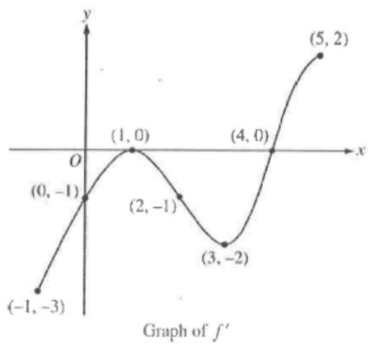
$\frac{dx}{dt} = \frac{22}{3}$

4. The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

(a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

(b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \le x \le 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \le x \le 5$? Show the analysis that leads to your answers.

(c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.



Graph of f'

a) PI @ $x = 1$ where f' goes from increasing to decreasing
PI @ $x = 3$ where f' goes from decr. to incr.

b) possible extrema where $f' = 0$ @ $x = 1, 4$
Absolute Min @ $x = 4$ where f' goes from negative to positive
f was decreasing until then since f' was negative and increasing after that where f' is +

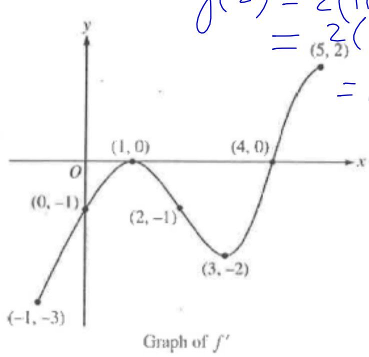
4. The figure above shows the graph of f', the derivative of the function f, on the closed interval $-1 \le x \le 5$. The graph of f' has horizontal tangent lines at $x = 1$ and $x = 3$. The function f is twice differentiable with $f(2) = 6$.

(a) Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

(b) At what value of x does f attain its absolute minimum value on the closed interval $-1 \le x \le 5$? At what value of x does f attain its absolute maximum value on the closed interval $-1 \le x \le 5$? Show the analysis that leads to your answers.

(c) Let g be the function defined by $g(x) = xf(x)$. Find an equation for the line tangent to the graph of g at $x = 2$.

$g(2) = 2(f(2))$
$= 2(6)$
$= 12$



Graph of f'

b) cont' Ab Max @ $x = 5$
f' never went from + to −
so f never goes from incr to decreasing. Highest point is the endpt.

c) given $(2, 6)$ $g(x) = x \cdot f(x)$
slope $\Rightarrow g'(x) = x f'(x) + (1) f(x)$
$g'(2) = 2 \cdot f'(2) + f(2)$
$= 2(-1) + 6$
$= 4$

$$\boxed{y - 12 = 4(x - 2)}$$

November 06, 2017

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

→ No hole or gap so must be a pointy place @ $x=2$

4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

(a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.

(b) On the axes provided, sketch the graph of a function that has all the characteristics of f.

a) Relative Max @ $x=2$
where f' changes from positive to neg indicating f changes from incr to decr.

b)